The Derivation of an Equation for Predicting Minimum Spouting Velocity

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The spouted bed developed by P. E. Gishler and K. B. Mathur is shown in operation with wheat as packing and air as the spouting medium in Figure 1. The photograph represents the bed in a split column which reveals the pattern of solids flow. In the patent (*1*) and also in their publication (*2*) these authors discussed the minimum fluid velocity necessary to spout a bed of solid particles and presented the following equation:

$$u_c = \left(\frac{D_p}{D_c}\right)\left(\frac{D_0}{D_c}\right)^{1/3} \sqrt{\frac{2gL(\rho_s - \rho_f)}{\rho_f}} \quad (1)$$

This equation is said to be developed by empirical methods with the component groups possibly having some theoretical significance.

An energy balance is put around the unit, as in Figure 1. The notation and datum lines are shown on the unit in Figure 2. The general energy balance for this system would then be

$$x_1 + \frac{u_1^2}{2g_c} + p_1v_1 + E_1 + Q - W$$

$$= x_2 + \frac{u_2^2}{2g_c} + p_2v_2 + E_2 \quad (2)$$

CONDITIONS ON EQUATION (2)

1. There is no heat lost or gained by the system between points 1 and 2.

$$Q = 0$$

2. There is no work given to or taken from the apparatus between points 1 and 2.

$$W = 0$$

3. The internal energy of the system is constant.

$$\Delta E = 0$$

ASSUMPTIONS

1. The specific volume at 1 and 2 are equal; there is no density change.

$$\rho_1 = \rho_2 = \rho_f$$

2. The fluid expands through the bed.
3. The average velocity at 2 is negligible compared with the average velocity at 1. For incipient spouting conditions this is not an erroneous assumption since $u_1 \cong$ 500 to 600 ft./sec. and $u_2 \cong$ 10 ft./sec.

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Incorporating these conditions and assumptions into Equation (2) results in

$$\frac{u_1^2}{2g_c} = x_2 + \frac{\Delta P}{\rho_f} \quad (3)$$

The bed depth, expressed in feet of flowing fluid, is quite small compared with the other terms in the equation. As an approximation it can be omitted.

Hence Equation (3) becomes

$$u_1 = C_s\sqrt{\frac{2g_c\,\Delta P}{\rho_f}} \quad (4)$$

which is readily recognized as an orifice type of equation.

Remembering that Equation (4) gives the velocity through the orifice, one can see that the following equation gives the superficial velocity through the column.

$$u_c = C_s\left(\frac{D_0}{D_c}\right)^2 \sqrt{\frac{2g_c\,\Delta P}{\rho_f}} \quad (5)$$

The equation that gives the pressure drop necessary to lift a bed or the fluidizing pressure drop as an approximation is

$$\Delta P = L(1 - \delta)(\rho_s - \rho_f) \quad (6)$$

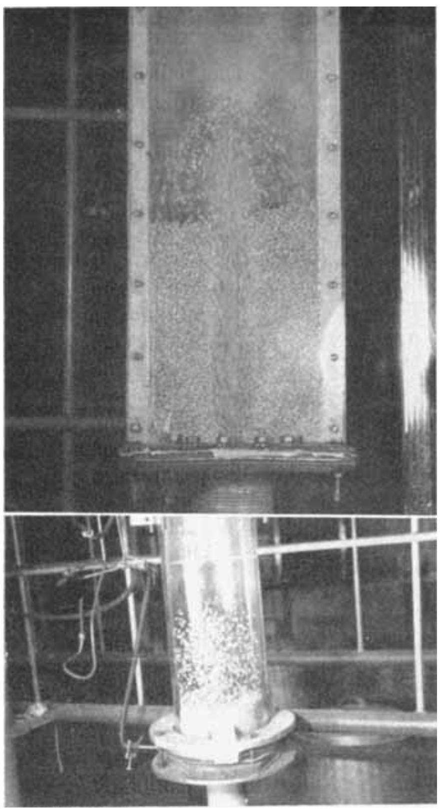

Fig. 1. Full and split columns.

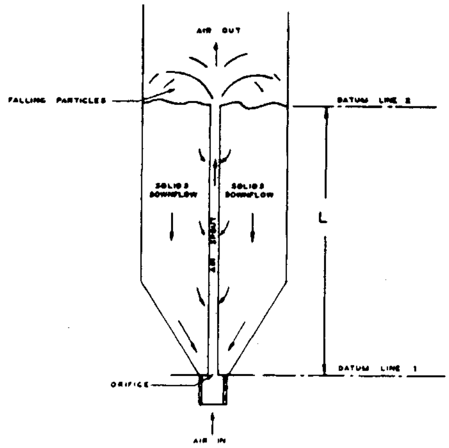

Fig. 2. Schematic diagram of a spouted bed.

If this equation is substituted into Equation (5), the following expression is obtained:

$$u_c = C_s\left(\frac{D_0}{D_c}\right)^2 \sqrt{\frac{2g_cL(1-\delta)(\rho_s - \rho_f)}{\rho_f}} \quad (7)$$

The pressure drops for spouting and fluidization differ from each other by a relatively constant amount. Hence any discrepancy in using the fluidizing pressure drop in Equation (7) should be revealed in the constant C_s. The similarity between the Gishler-Mathur equation and Equation (7) is obvious.

NOTATION

v = specific volume, cu. ft./lb.
u = velocity, ft./sec.
D = diameter, ft.
g_c = gravitational constant, 32.2 ft./sec.^{2}
L = depth of bed, ft.
X = distance from arbitrary datum to system ft.-lb./lb.
E = internal energy, ft.-lb./lb.
Q = heat, ft.-lb./lb.
W = work, ft.-lb./lb.
C_s = correlating coefficient
p = static pressure, lb./sq. ft.
x_2 = bed depth, ft. of flowing fluid

Greek Symbols

ρ = density, lb./cu. ft.
Δ = difference
δ = void fraction

Subscripts

c = column
0 = orifice
s = solid, spouting
f = fluid
p = particles
1 = position at orifice
2 = position at top of bed

LITERATURE CITED

1. Gishler, P. E., and K. B. Mathur, U. S. patent 2,786,280.
2. *Ibid., A.I.Ch.E. Journal,* **1,** 157 (1955).